A VIDEO COMPRESSION SCHEME WITH OPTIMAL BIT ALLOCATION BETWEEN DISPLACEMENT VECTOR FIELD AND DISPLACED FRAME DIFFERENCE

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ABSTRACT

In this paper, we address the fundamental problem of optimally splitting a video sequence into two sources of information, the displaced frame difference (DFD) and the displacement vector field (DVF). We first consider the case of a lossless motion compensated video coder (MCVC) and derive a general Dynamic Programming (DP) formulation which results in an optimal tradeoff between the DVF and the DFD. We then consider the more important case of a lossy MCVC and present an algorithm which solves optimally the bit allocation between the rate and the distortion. This algorithm is based on Lagrangian relaxation and the DP approach introduced for the lossless MCVC. We then present an H.263-based MCVC which uses the proposed optimal bit allocation scheme and compare its results to H.263. As expected, the proposed coder is superior in the rate-distortion sense.

1. INTRODUCTION

Video compression attracted considerable attention over the last decade [1]. There is a large redundancy in any video sequence which has to be exploited by every efficient video coding scheme. This redundancy is divided into temporal and spatial. The temporal redundancy is usually reduced by motion compensated prediction of the current frame from a previously reconstructed frame, whereas the spatial redundancy left in the prediction error is commonly reduced by a transform coder or a vector quantizer. Video coders which use the concept of motion compensated prediction are henceforth called motion compensated video coders (MCVC). All existing video standards belong to this class of video coders. In an MCVC, the original video sequence is represented by the displacement vector field (DVF) and the displaced frame difference (DFD). A fundamental problem of MCVC is the bit allocation between the DFD and the DVF. In this paper we present a general theory which uses operational rate-distortion curves to solve this problem for a finite set of admissible quantizers and motion vectors.

There have been previous attempts to solve the optimal tradeoff between DVF and DFD. In [2], the authors assume a stochastic model for the distribution of the DFD and proceed to calculate the entropy of a given block based on some observed statistics. This entropy is then used to decide if a bock should be split into four smaller blocks with their own motion vectors, or if the block should be kept as a basic unit.

In [3], the problem of rate-constrained motion estimation is considered and the optimal bit allocation condition for a strictly convex and everywhere differentiable multivariate rate distortion function is derived. It is applied to the problem of optimal bit allocation between the DVF and the DFD and a rate-constrained, region based motion estimator is introduced. In this paper, we do not assume knowledge of a convex and everywhere differentiable multivariate rate distortion function, but instead we deal with a set of finite quantizers and motion vectors.

In [4], a variable block size motion estimator is presented. It is implied that the motion vectors are encoded by pulse code modulation (PCM) and hence the resulting optimization procedure is straightforward. In contrast to this work, we allow for more sophisticated encoding schemes of the DVF, such as, the popular differential PCM (DPCM).

The paper is organized as follows: In section 2. we define the problem under consideration. In section 3. we derive the optimal solution for a lossless MCVC. This solution is then extended in section 4. to include lossy MCVC. In section 5. we develop a lossy video coder based on the presented theory and in section 6. we discuss some implementation issues which reduce the computational complexity of the presented coder. The experimental results of this coder are presented in section 7. and the paper is summarized in section 8..

2. NOTATION AND ASSUMPTIONS

Our study of the optimal bit allocation between the DVF and DFD is restricted to the frame level. In other words we do not attempt to optimally allocate the bits among the different frames of a video sequence. The reader interested in that problem is referred to [5]. For the rest of this paper we assume that a rate control algorithm has given us the maximum number of bits available (R_{max}) or the maximum acceptable distortion (D_{max}) for a given frame.

We assume that the current frame is segmented into N regions, $o_1, \ldots, o_N$, and that this segmentation and the associated scanning path are known to both the encoder and the decoder. We then number the regions such that the scanning path visits them in ascending order. Every region o_i has a motion vector $m_i \in M_i$, and a quantizer $q_i \in Q_i$ associated with it, where M_i is the set of all admissible motion vectors for region o_i and Q_i is the set of all admissible quantizers for region o_i. As in every practical video coding scheme, we assume that the sets M_i and Q_i are finite. Let us define a decision vector $v_i = [m_i, q_i] \in V_i$, for every region o_i which contains the motion vector and the quantizer for that region. $V_i = M_i \times Q_i$ is the admissible decision vector set for region o_i. We assume that the frame distortion $D_k(v_1, \ldots, v_N)$ is a function of the current frame, the previously reconstructed frame, and the decision vectors $v_1, \ldots, v_N$. Equivalently, we assume the same dependency for the frame rate $R_k(v_1, \ldots, v_N)$. Note that the term "frame rate" represents the number of bits used to encode a certain frame and not the number of frames per second.

The next assumption expresses the idea that the frame rate and frame distortion can be decomposed into a sum of region rates $r_i(v_{i-a}, \ldots, v_{i+b})$ and region distortions $d_i(v_{i-a}, \ldots, v_{i+b})$, which only depend on a local neighborhood. We assume that there exist integers $a \geq 0$, and $b \geq 0$ and a family of functions d_i and r_i such that the following holds,

$$D_k(v_1, \ldots, v_N) = \sum_{i=1}^{N} d_i(v_{i-a}, \ldots, v_{i+b}), \quad (1)$$

$$R_k(v_1, \ldots, v_N) = \sum_{i=1}^{N} r_i(v_{i-a}, \ldots, v_{i+b}), \quad (2)$$

where the decision vectors v_j not belonging to any region ($j \notin [1, \ldots, N]$), represent the boundary parameters and can be set to any desired value.

It is noted here that assumptions (1) and (2) are quite general and valid for every existing video coding standard. Also most of the commonly used distortion measures, such as the mean squared error (MSE) or the peak signal to noise ratio (PSNR), satisfy assumption (2).

3. LOSSLESS MCVC

Since for a lossless MCVC the reconstructed frame is identical to the original frame, the frame distortion will be zero and the goal is to minimize the number of bits required for the DVF and the DFD. This can be stated as follows,

$$\min_{v_1, \ldots, v_N} R_k(v_1, \ldots, v_N). \quad (3)$$

Since this is a lossless MCVC, the DFD is not quantized, but encoded losslessly. With a slight abuse of notation let q_i represent different lossless encoding schemes for region o_i (i.e., DPCM with different predictor order, etc.), instead of different quantizers. Since we will refer to this algorithm later on, we will still call the q_i's quantizers in the following derivation.

As stated in assumption (2), the frame rate R_k is the sum of rates which only depend on local neighborhoods. We will now employ this assumption to derive a Dynamic Programming (DP) [6] solution to problem (3). We will use generic terms, ($g_i(v_{i-a}, \ldots, v_{i+b})$ and $G_k(v_1, \ldots, v_N)$), in this derivation since we will refer to it later on. Let

$$g_i(v_{i-a}, \ldots, v_{i+b}) = r_i(v_{i-a}, \ldots, v_{i+b}), \quad (4)$$

$$G_k(v_1, \ldots, v_N) = \sum_{i=1}^{N} g_i(v_{i-a}, \ldots, v_{i+b}), \quad (5)$$

$$g_l^*(v_{l-a+1}, \ldots, v_{l+b}) = \min_{v_1, \ldots, v_{l-a}} \sum_{i=1}^{l} g_i(v_{i-a}, \ldots, v_{i+b}). \quad (6)$$

From Eq. (6) it follows that,

$$\begin{aligned} & g_{l+1}^*(v_{l+1-a+1}, \ldots, v_{l+1+b}) \\ &= \min_{v_1, \ldots, v_{l+1-a}} \sum_{i=1}^{l+1} g_i(v_{i-a}, \ldots, v_{i+b}) \quad (7) \\ &= \min_{v_{l+1-a}} \left[\min_{v_1, \ldots, v_{l-a}} \left(\sum_{i=1}^{l} g_i(v_{i-a}, \ldots, v_{i+b}) + \right. \right. \\ & \left. \left. g_{l+1}(v_{l+1-a}, \ldots, v_{l+1+b}) \right) \right] \quad (8) \end{aligned}$$

Since $g_{l+1}(v_{l+1-a}, \ldots, v_{l+1+b})$ does not depend on $v_1, \ldots, v_{l-a}$, it can be moved outside the inner minimization. Then the resulting inner minimization is equal to $g_l^*(v_{l-a+1}, \ldots, v_{l+b})$ in Eq. (6) and the following DP recursion formula results,

$$\begin{aligned} & g_{l+1}^*(v_{l+1-a+1}, \ldots, v_{l+1+b}) = \quad (9) \\ & \min_{v_{l+1-a}} \left[g_l^*(v_{l+1-a}, \ldots, v_{l+b}) + g_{l+1}(v_{l+1-a}, \ldots, v_{l+1+b}) \right]. \end{aligned}$$

Forward DP (also called the Viterbi algorithm) can now be used to solve problem (3). The time complexity for the DP approach depends directly on the size of the neighborhood and is $O(N * |V_i|^{a+b+1})$, where we assume that all V_i's have the same cardinality $|V_i|$.

4. LOSSY MCVC

In this section we study the case of lossy MCVC. Clearly for a lossy MCVC it does not make sense to minimize the frame rate R_k with no additional constraints, since this would lead to a very high frame distortion D_k.

The most common approach to solve the tradeoff between the frame rate and the frame distortion is to minimize the frame distortion D_k subject to a given maximum frame rate R_{max}. Clearly since the total number of regions N is known, minimizing the total distortion is equivalent to minimizing the average distortion. This problem can be formulated in the following way,

$$\min_{v_1, \ldots, v_N} D_k(v_1, \ldots, v_N), \text{ s.t.: } R_k(v_1, \ldots, v_N) \leq R_{max}. \quad (10)$$

This constrained discrete optimization problem is very hard to solve in general. In fact the approach we propose will not necessarily find the optimal solution but only the solutions which belong to the convex hull of the rate-distortion curve. On the other hand, as we show in Sec. 7., the solutions on the rate-distortion curve tend to be quite dense and hence the convex hull approximation is very good.

We solve this problem using the concept of Lagrangian relaxation [7, 8], which is a well known tool in operations research. First we introduce the Lagrangian cost function which is of the following form,

$$J_\lambda(v_1, \ldots, v_N) = D_k(v_1, \ldots, v_N) + \lambda * R_k(v_1, \ldots, v_N), \quad (11)$$

where $\lambda \geq 0$ is called the Lagrangian multiplier. It has been shown in [7, 8, 9] that if there is a λ^* such that,

$$[v_1^*, \ldots, v_N^*] = \arg \min_{v_1, \ldots, v_N} J_{\lambda^*}(v_1, \ldots, v_N), \quad (12)$$

leads to $R_k(v_1^*, \ldots, v_N^*) = R_{max}$, then $v_1^*, \ldots, v_N^*$ is also an optimal solution to (10). It is well known that when λ sweeps from zero to infinity, the solution to problem (12) traces out the convex hull of the rate distortion curve, which is a non-increasing function. Hence bisection could be used to find λ^*. A faster converging algorithm which uses the convexity of the curve is employed in [10]. Therefore the problem at hand is to find the optimal solution to problem (12). We next show how the original DP approach can be modified to find the global minimum of problem (12). For a given λ, let the $g_i(v_{i-a}, \ldots, v_{i+b})$ functions be defined as follows,

$$g_i(v_{i-a}, \ldots, v_{i+b}) = r_i(v_{i-a}, \ldots, v_{i+b}) + \lambda * d_i(v_{i-a}, \ldots, v_{i+b}), \quad (13)$$

which implies that, $G_k(v_1, \ldots, v_N) = J_\lambda(v_1, \ldots, v_N)$. Hence the DP algorithm presented in section 3. leads to the optimal solution of problem (12).

5. A VIDEO COMPRESSION SCHEME WITH OPTIMAL BIT ALLOCATION BETWEEN DVF AND DFD

We apply the presented theory to the optimal allocation of bits between the DFD and the DVF for a video coder which is largely based on the ITU standard for very low bit rate video coding H.263. In fact the proposed coder is almost identical to test model 4 (TMN4) [11] of H.263 with some noteworthy exceptions which we will point out later on. We use the peak signal to noise ratio (PSNR) as the distortion measure. It is by, $D_k = \frac{255^2}{\mathrm{MSE}(\tilde{f}_k, f_k)}$, where $\mathrm{MSE}(\tilde{f}_k, f_k)$ is the mean squared error between the luminance channel of the reconstructed and the original frames. In the case of TMN4, the PSNR frame distortion can be written as,

$$D_k(v_1, \ldots, v_N) = \sum_{i=1}^{N} d_i(v_i), \qquad (14)$$

since the region distortion $d_i(v_i)$ only depends on the selected motion vector and the selected quantizer for that region. We use "quarter common intermediate format" (QCIF) sequences, which have dimensions 176×144 pixels. Since TMN4 breaks the frame into 11×9 macro blocks of size 16×16, the regions $o_1, \ldots, o_N$ are defined to be these macro blocks. Clearly, $N = 99$ for this implementation. TMN4 uses a vector median of three neighboring macro blocks to predict the current motion vector. Since the efficiency of DP depends on the order of the dependency, a first order DPCM scheme is used, where the previous motion vector is used to predict the current one and instead of raster scan a Hilbert scan is used [12]. Hence the total frame rate can now be expressed by,

$$R_k(v_1, \ldots, v_N) = \sum_{i=1}^{N} r_i(v_{i-1}, v_i), \qquad (15)$$

where

$$r_i(v_{i-1}, v_i) = r_i^{QDFD}(v_i) + r_i^{DVF}(v_{i-1}, v_i), \qquad (16)$$

$r_i^{QDFD}(v_i)$ are the bits needed to encode the DFD of block o_i using the quantizer q_i and the motion vector m_i, and $r_i^{DVF}(v_{i-1}, v_i)$ the bits needed to encode the motion vector difference $(m_i - m_{i-1})$. Let $e_i \in E_i$ be the encoding mode of macro block o_i, where $E_i = \{$Intra, Inter, Skip, Prediction$\}$. The encoding mode can be set differently for each macro block. The first three modes are the same as in TMN4, and for the Prediction mode introduced here, only the motion vector is sent. Let $QP_i \in Z_i$ be the DCT domain quantizers for block o_i, where Z_i is the set of all admissible DCT domain quantizers for block o_i. In TMN4, 31 different DCT domain quantizers are admissible. Note the distinction made between quantizers and DCT domain quantizers. The reason for this is that the modes can be considered quantizers too. Therefore we can define the new set of quantizers for block o_i as $q_i = [e_i, QP_i] \in Q_i$ where $Q_i = E_i \times Z_i$.

In TMN4 a quantizer is selected by transmitting a quantizer step size QP. QP is encoded using a modified delta modulation with a range of ± 2. Hence the quantizer step size of block o_i, QP_i, is equal to $QP_{i-1} + \delta_i$, where $\delta_i \in [-2, -1, 0, 1, 2]$. At the beginning of the frame, QP_1 of the first block is coded using PCM. Clearly, this delta modulation introduces another dependency which can be captured by modifying $r_i(v_{i-1}, v_i)$ from Eq. (16), to

$$r_i(v_{i-1}, v_i) = r_i^{QDFD}(v_i) + r_i^{DVF}(v_{i-1}, v_i) + r_i^{QP}(v_{i-1}, v_i), \qquad (17)$$

where $r_i^{QP}(v_{i-1}, v_i)$ corresponds to the bits needed to encode $\delta_i \in [-2, -1, 0, 1, 2]$. $r_i^{QP}(v_{i-1}, v_i)$ is set to infinity for a QP_i which is out of reach.

6. IMPLEMENTATION ISSUES

From a theoretical point of view, every possible motion vector of block o_i should be included in the set of admissible motion vectors M_i. Most of these motion vectors are not likely candidates for the optimal path since they do not correspond well to the real motion in the scene and therefore they lead to a high distortion and a high rate. We propose the following strategy to constrain the set M_i. An initial motion vector is first found using block matching with integer accuracy and the sum of absolute error matching criterion. Then the set M_i is defined as the set which contains this motion vector plus the K neighboring motion vectors at half pixel locations. This leads to $|M_i| = K + 1$. Our experiments have shown (see Section 7.) that a $K = 8$ results in a negligible performance loss.

A similar situation arises for the quantizer selection. In TMN4, the quantizer parameter QP_i can take on values between 1 and 31. Since a nearly constant distortion is usually targeted, a reduced admissible quantizer set, which is centered around the quantizer step size which leads to the desired quality, can be used without any noticeable loss of performance. The set employed in the presented experiments is $Z_i = \{8, 9, 10, 11, 12\}$. Further reductions in complexity and a fast evaluation of the rate distortion function are discussed in [12] and [10].

7. EXPERIMENTS

Note that the presented coder, like TMN4, writes a bit stream which is uniquely decodable by our decoder. Hence the listed bit rates are the effective number of bits used and not an estimate of the entropy.

In order to compare TMN4 and the proposed coder, TMN4 was used to encode every 4^{th} frame of the first 200 frames of the QCIF color sequence "Mother and Daughter" with a fixed quantizer step size $QP = 10$. The first frame was Intra coded using the same quantizer step size. Since the "Mother and Daughter" sequence is considered to be recorded at 30 frames/second, this leads to an encoded rate of 7.5 frames/second. The resulting frame rate profile (average: 23.4 kbits) and frame distortion profile (average: 33.0 dB) were used for the comparison between TMN4 and the proposed coder.

The proposed coder is compared to TMN4 in the case where their frame rates are matched. This can be achieved by setting R_{max}, the maximum frame rate from Eq. (10) equal to the frame rate obtained by TMN4. Clearly R_{max} changes from frame to frame, following the rate profile of TMN4 and the proposed coder will minimize the resulting frame distortion for the given frame rate. The resulting rate profile is equivalent to the TMN4 rate profile since for every frame the proposed coder uses the same number of bits as TMN4. The proposed coder results in a reconstructed sequence with an average distortion of 33.5 dB, which is half a dB better than TMN4, and of higher visual quality. Besides being able to outperform TMN4 in the rate distortion sense, this experiment also shows the enormous potential of this approach with respect to rate control since the proposed coder can follow any arbitrary bit rate profile. The optimal encoding mode selection, the optimal quantizer selection and the optimal DVF are displayed in Figs. 1, 2 and 3 for the 16^{th} frame of the "Mother and Daughter" sequence, respectively. Note in Fig. 1 how the new object (hand) and the uncovered areas (left of the hand) are Intra coded and the stationary background is replaced by the blocks from the previously decoded frame (Skip mode).

Also note the smoothness of the motion vector field in Fig. 3, which can be encoded very efficiently by DPCM.

8. SUMMARY

We have presented a general theory for the optimal bit allocation between displacement vector field (DVF) and displaced frame difference (DFD). The theory can be applied to all region based motion compensated video coders (MCVC), which includes all current video standards. We first considered a lossless MCVC and derived the optimal bit allocation algorithm which is based on dynamic programming (DP). We then addressed the problem of lossy MCVC and we showed that Lagrangian relaxation and DP can find the convex hull approximation to the optimal solution. We finally presented a video coder which is largely based on H.263, and uses this optimal bit allocation between the DVF and the DFD. The presented results show that the proposed coder is superior to H.263 in the rate distortion sense.

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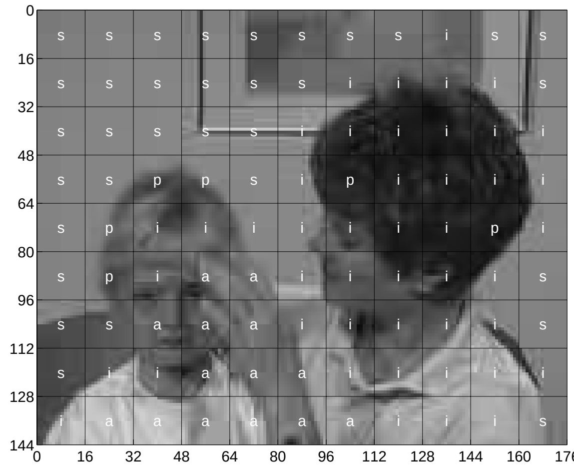

Figure 1. (i) Inter, (s) Skip, (p) Pred., (a) Intra.

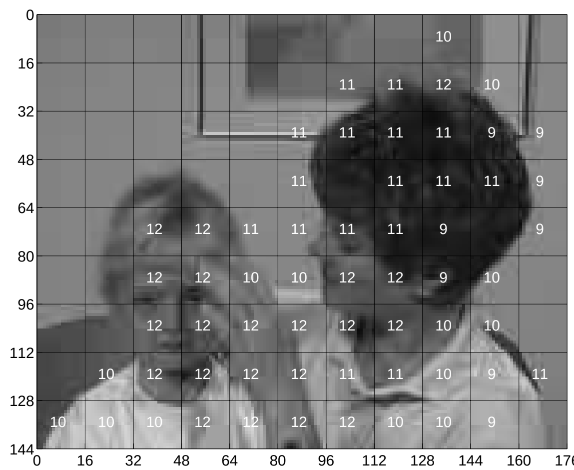

Figure 2. The optimal quantizer selection.

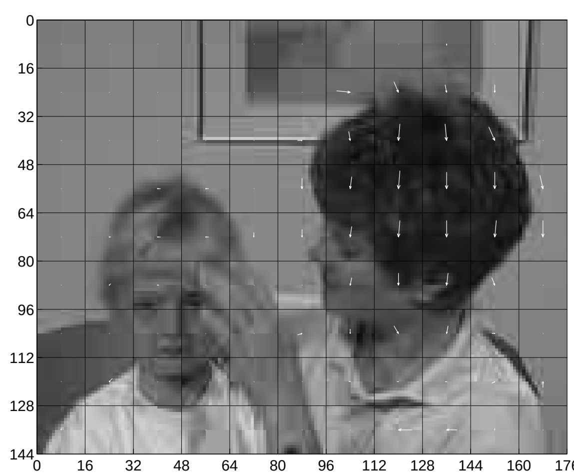

Figure 3. The optimal motion vector field.

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