Differential Detection of GMSK Signals with Low B_tT Using the SOVA

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***Abstract*—A noncoherent Gaussian minimum phase-shift keying (GMSK) detector using differential phase detection combined with the soft-output Viterbi algorithm (SOVA) is presented. This approach overcomes the severe intersymbol interference (ISI) of GMSK signals with low B_tT. Unlike conventional detectors the SOVA produces soft-decision bits resulting in larger coding gains in subsequent convolutional decoders.**

***Index Terms*— GMSK, noncoherent detection, soft decision, SOVA.**

I. Introduction

THE COMBINATION of constant envelope and high spectral efficiency makes the Gaussian minimum-shift keying (GMSK) modulation scheme highly popular. Low B_tT makes the modulation scheme even more bandwidth-efficient at the price of increased intersymbol interference (ISI). Incoherent receivers often are the architecture of choice if dealing with fading channels. One of the more popular incoherent receiver structures applies a differential phase detector (DPD) [1]. DPD on its own produces poor results for low B_tT. Some improvement is possible using 2- or 3-b DPD [1]. Techniques employing feedback of previously decided symbols (DF) only cancel ISI caused by past symbols, while half of the ISI is, however, caused by symbols still to follow. If this part of ISI is to be canceled, too, the symbol decisions become mutually dependent in both causal and noncausal time directions. This situation can be resolved by maximum-likelihood sequence estimator (MLSE) using the Viterbi algorithm (VA). Coding gains of convolutional codes are higher if soft-decisions (SD's) of the channel bits are available at the input to the decoder [2]. The VA generally produces hard-decision output. Soft-decision output is provided by the soft-output Viterbi algorithm (SOVA) [3], [4], which is used in the architecture presented.

II. The GMSK Modulation Scheme

The baseband representation of a GMSK signal at the input of the receiver in an additive white Gaussian noise (AWGN) channel can be expressed as

$$s(t) = A \exp\left[j2\pi \sum_{i=0}^{k} \alpha_i \phi(t - iT)\right] + n_i, \qquad kT < t < (k+1)T \quad (1)$$

with α_i being the data symbols $\in \{+1, -1\}$ and the phase function

$$\phi(t) = \int_{-\infty}^{t} g(\tau)\, d\tau. \quad (2)$$

$g(t)$ is the Gaussian pulse form and n_i is an AWGN component. The DPD performs the following operation:

$$d(mT_s) = s(mT_s) \cdot s^*(mT_s - T_D) \quad (3)$$

where $(.)^*$ is the complex conjugate operation. Neglecting the noise component n_i and inserting (1) and (2) into (3) we get

$$d(mT_s) = A^2 \exp\left[j2\pi \sum_{i=0}^{k} \alpha_i \int_{mT_s-(i+1)T}^{mT_s-iT} g(\tau)\, d\tau\right]. \quad (4)$$

The best sampling point for the estimation of α_i is when the decision variable $d(mT_s)$ is most influenced by α_i. This is the case when the integral is maximized for a given i

$$\max_{mT_s} \int_{mT_s-(i+1)T}^{mT_s-iT} g(\tau)\, d\tau = \int_{-(T/2)}^{T/2} g(\tau)\, d\tau. \quad (5)$$

The best sampling point for the ith symbol is therefore $mT_s = (i + (1/2))T$. Since $\int_{-\infty}^{\infty} g(\tau) d\tau = (1/4)$ the DPD fails to work if $\int_{-(T/2)}^{T/2} g(\tau)\, d\tau < (1/8)$, which is the case for $B_tT < 0.195$. At this point the combined influence of symbols adjacent to a symbol becomes larger than the phase change initiated by the symbol under detection.

III. DPD Combined with VA

For the following extension of the receiver the slicer is replaced by the VA. Because of the non-Gaussian nature of the noise after the differential demodulator, squared Euclidean distances do not lead to maximum-likelihood sequence estimation. Nonetheless, Euclidean distances are simple measures and give good results. The extended receiver has been simulated in an AWGN channel without fading. The bandwidth of the receive filter[1] has been chosen[2] as $BT = 1$, so that $\mathrm{SNR} = E_b/N_o$. Fig. 1 shows the bit-error rate (BER) performance of DPD with and without VA. The BER curve for coherent reception has been taken from Yongaçoḡlu *et al.* [1] and is included for comparison. The VA improves the result of the DPD for $B_tT = 0.25$ by 7–8 dB. For an error probability of 1% an E_b/N_0 of about 8.5 dB is needed, whereas the DPD alone needs at least 15 dB. The performance of the DPD–VA is only 2–3 dB worse than coherent detection and is comparable with the combined 2- and 3-b differential detectors with decision feedback [1]. Even for $B_tT = 0.15$

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[1] Note that this BT of the receive filter is different from B_tT chosen for the premodulation filter.

[2] The receive filter with $BT = 1$ is inherently included in the simulation if the sampling rate is equal to the symbol rate. This overcomes problems with ISI introduced by the receive filter [1].

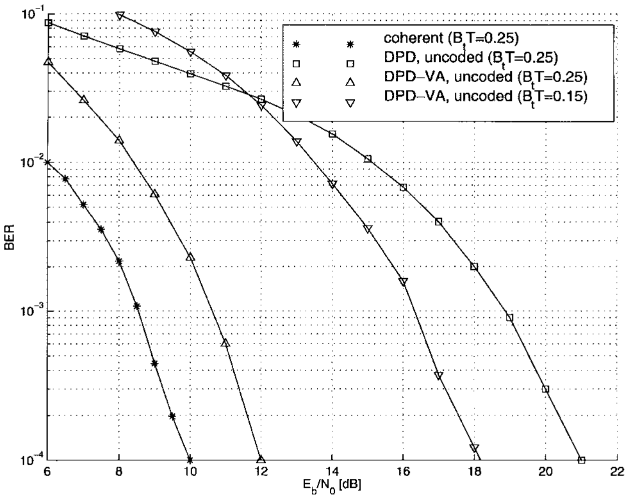

Fig. 1. BER performance of GMSK detected with DPD–VA (uncoded) on an AWGN channel.

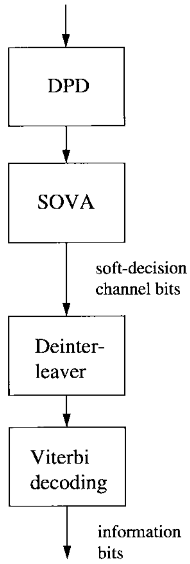

Fig. 2. Block diagram of part of the extended receiver including the SOVA.

(where DPD is impossible) the DPD–VA delivers acceptable results. A 1% BER results with $E_b/N_0 = 13.5$ dB.

IV. The SOVA

A. Theory

The previously described VA produces hard-decision output which is sufficient if no error control coding is applied. If a convolutional code is employed, it is desirable to have soft decisions available at the input to the decoder on the grounds of higher coding gain. For a symbol-by-symbol-based detector, the soft decisions are often easily derived from the likelihood function of individual symbols. This is not possible with the VA, since the decisions are based on sequences of symbols rather than on individual symbols. Hagenauer/Hoeher [3] and Berrou *et al.* [4] suggest a scheme (or rather several variants of one basic algorithm) where the accumulated metric of a survivor path is compared with the accumulated metric of the concurrent path associated with a particular state. This difference then serves as a soft-decision value, provided the final path decision involves this particular state. Otherwise, the information is discarded and some other differences related to another state is used. This algorithm is referred to as the basic weighting algorithm [4]. The shortcoming of this algorithm is that high weights do not always reflect high reliability. In fact, the situation may occur where a survivor path is just about superior to a concurrent path with high weights back on that partial path, misleading to assume that those symbols are reliable. An improvement has been proposed [4], based on probabilistic calculations by Battail [5], where weights of previous states (j time steps back) are updated as a function of weights of present states m at time k according to

$$\begin{aligned} a_j(k, m) = {} & \log\{\exp[a'_j(k, m)] + \exp[a_j(k, m) + a'_j(k, m)] \\ & + \exp[a_j(k, m) + |a(k, m)|] \\ & + \exp[a_j(k, m) + a'_j(k, m) + |a(k, m)|]\} \\ & - \log\{1 + \exp[a_j(k, m)] + \exp[|a(k, m)|] \\ & + \exp[a'_j(k, m) + |a(k, m)|]\} \end{aligned} \tag{6}$$

where $a(k, m)$ is the difference of the accumulated path metric multiplied with the symbol value $s_j(k, m)$. $a(k, m)$ therefore represents the weighted decision of the present state. $a_j(k, m)$ and $a'_j(k, m)$ are the weighted decisions memorized j time steps back at the corresponding state of the survivor path and concurrent path, respectively. The maximum j at which the update has to take place is determined by the first state at which the concurrent path and the survivor path differ (one state after divergence). Equation (6) is rather impractical for implementation. Berrou *et al.* [4] simplify (6) without much impact on the BER performance to

$$a_j(k, m) = s_j(k, m) \cdot \min[|a_j(k, m)|, |a(k, m)|] \tag{7}$$

for the case when the symbol decisions of the survivor path and concurrent path differ. When the symbol decisions of the survivor and concurrent path are equal, the simplified update formula of (6) involves the weighted decision of the concurrent path $a'_j(k, m)$. However, in this case the update is not important, therefore not necessary. This way, the weight of the concurrent path $a'_j(k, m)$ is no longer needed. This is the algorithm implemented within the VA after the DPD to detect GMSK signals. VA detected signals generally exhibit error bursts. In order not to limit the performance of the second code an interleaver is usually deployed in concatenated systems. The extended part of the receiver is shown in Fig. 2.

B. BER Performance

Simulations of the receiver as shown in Fig. 2 have been carried out. The interleaving depth was chosen as $d = 50$. This interleaver operates on a frame length of 1000 b. The convolutional code used is a nonsystematic rate 1/2 code with constraint length $K = 3$ and the generator polynomials $g_1(X) = X^2 + X + 1$ and $g_2(X) = X^2 + 1$. The BER performance of the SOVA combined with the coding mentioned is shown in Fig. 3. Coded BER's obtained with plain differential detection are included for comparison. E_b represents the bit energy per information bit, which is 3 dB more than the

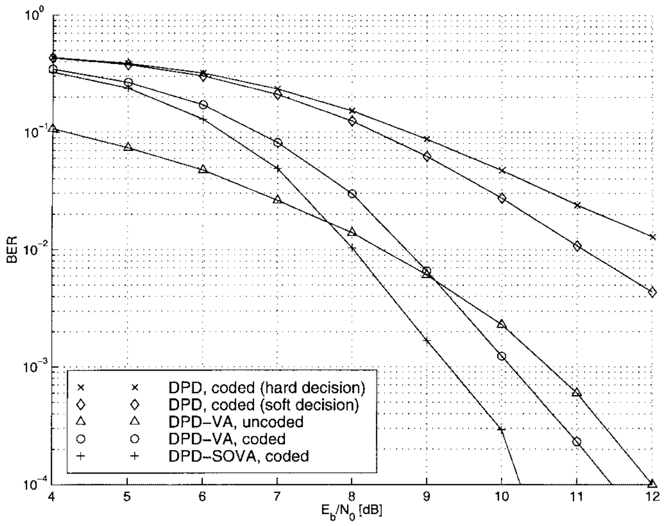

Fig. 3. BER performance of GMSK detected with DPD–SOVA $B_tT = 0.25$ on an AWGN channel.

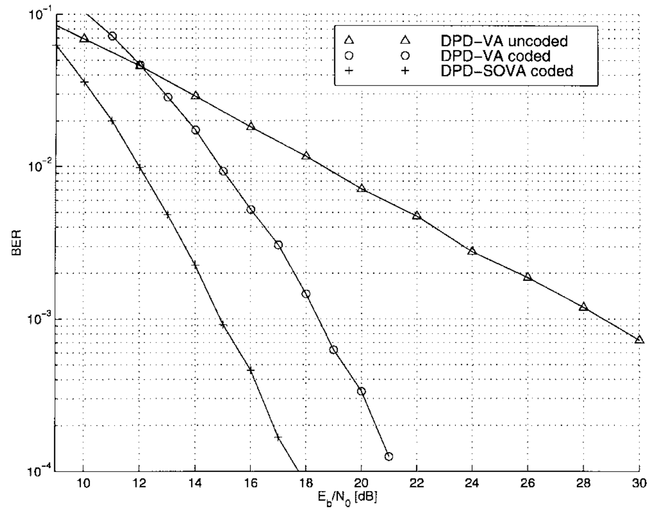

Fig. 4. BER performance of GMSK detected with DPD–SOVA $B_tT = 0.25$ on a flat-fading channel.

bit energy per channel bit for the coded bits. Fig. 3, which shows the BER performance of the SOVA with subsequent coding on an AWGN channel, therefore reflects "real" coding gain. The error control code used in the simulation shows the principle of how soft decisions may help to increase the coding gain. Increasing the constraint length of the convolutional code will increase the coding gain further. With the current configuration, the coding gain of the rate 1/2 code with constraint length $K = 3$ at a BER of 10^{-3} is only marginal for hard-decision decoding. Soft-decision, however, results in about 1.5-dB gain over the uncoded signal. A much larger gain is obtained under a fading channel. Simulations in a flat Rayleigh-fading channel resulted in a BER performance as shown in Fig. 4. Coding gains are now around 14 dB at a BER of 10^{-3}, outperforming the hard-decision VA by more than 3 dB.

V. Conclusions

Although the use of differential phase detection combined with the VA to detect a sequence of GMSK symbols leads to a more complex receiver than the approach using differential phase detection with feedback, the advantage of the former is the possibility of producing soft decisions. This leads to a further improvement of about 1–3 dB (depending on the channel) compared to hard decisions if appropriate error control codes are used.

The present work has outlined the use of the VA to differentially detect GMSK signals with low B_tT. An extension to the VA has been described giving soft-decision output (SOVA). Using the SOVA enables combined modulation/coding schemes to make very efficient use of the spectrum while staying constant-envelope. Simulations showed the efficacy of the proposed receiver for $B_tT = 0.25$ in an AWGN and a flat Rayleigh-fading channel. The detection method described in this paper is not limited to GMSK signals but can be applied to any continuous-phase modulation scheme.

References

[1] A. Yongaço̅glu, D. Makrakis, and K. Feher, "Differential detection of GMSK using decision feedback," *IEEE Trans. Commun.*, vol. 36, pp. 641–649, June 1988.
[2] E. R. Berlekamp, "The construction of fast, high-rate, soft decision block decoders," *IEEE Trans. Inform. Theory*, vol. IT-29, pp. 372–377, May 1983.
[3] J. Hagenauer and P. Hoeher, "A Viterbi algorithm with soft-decision outputs and its applications," in *Proc. GLOBECOM'89*, vol. 3, Dallas, TX, Nov. 1989, pp. 1680–1686.
[4] C. Berrou, P. Adde, E. Angui, and S. Faudeil, "A low complexity soft-output Viterbi decoder architecture," in *Proc. ICC'93*, Geneva, Switzerland, May 1993, pp. 737–740.
[5] G. Battail, "Pondération des symboles décodés par l'algorithme de Viterbi," *Ann. Telecommun.*, vol. 42, pp. 31–38, Jan. 1987.